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RESOLUTION OF A FAMILY OF LAGRANGE EQUATIONS

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Abstract

The article deals with a family of Lagrange equations, indexed by a parameter $q \in \mathbb{Z}$:

$$x^2 - (q^2 - 4) y^2 = 4q.$$

Owing to $q \in \mathbb{Z}$, we identify the cases where these equations have solutions. They are obtained thanks to an unusual method. We show how acnodal cubic curves appear in this context.

Keywords and phrases: Lagrange equation, Pell-Fermat equation, acnode curve.

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References

[1] T. Andreescu and D. Andrica, Quadratic Diophantine Equations, Springer-Verlag, 2013.

[2] B. D. Bitim and R. Keskin, On some Diophantine equations, J. Inequal. Appl. 1(162) (2013), 1-12.

[3] J.-P. Bode and H. Harborth, Positive integers $(a^2 + b^2)/(ab + 1)$ are squares, Applications of Fibonacci Numbers, Vol. 9, Kluwer Acad. Publ., Dordrecht, 2004, pp. 63-67.

[4] J. Campbell, A solution to 1988 IMO QUESTION 6 (The Most Difficult Question Ever Set at an IMO), Math. Competit. 1 & 2 (1988), 29-32.

[5] M. Hirschhorn, A note on the Pell equation $x^2 - (q^2 - 4) y^2 = 1$, paper 86. web.math.unsw.edu.au/

[6] F. Luca, C.F. Osgood and P. G. Walsh, Diophantine approximations and a problem from the 1988 IMO, Rocky Mount. J. Math. 36 (2006), 637-648.

[7] W. J. LeVeque, Topics in Number Theory, Vols. 1 and 2, Dover Publications, Inc., 1984.

[8] K. Matthews, The Diophantine equation $x^2 - Dy^2 = N$,

www.numbertheory.org/php/pell4.html

[9] P. Nicaise, Les Courbes Algebriques Planes du Troisieme Ordre, EPU Publibook, 2012.

[10] N. J. A. Sloane, The On-line Encyclopedia of Integer Sequences, https://oeis.org/

[11] N. Tzanakis, Effective solution of two simultaneous Pell equations by the Elliptic Logarithm Method, Acta Arith. 103 (2002), 119-135.

[12] N. Tzanakis, Solving elliptic Diophantine equations by estimating linear forms in elliptic logarithms, The case of quartic equation, Acta Arith. 75 (1996), 185-190.