REAL CRITICAL LEVEL LINES OF A POLYNOMIAL IN THE PLANE

Alexander D. Bruno and Alexander B. Batkhin

Abstract

We propose a method for investigating the global problem on the location of all level lines of a real polynomial on the real plane. For this purpose it is necessary to compute its critical points and critical curves, and then the critical values of the polynomial (a finite number of them). All critical level lines and one representative of non-critical level lines corresponding to intervals of polynomial values between neighboring critical values are calculated using them. For critical values, the study is based on the methods of power geometry. Software for these calculations is specified. Two nontrivial examples are considered. Sometimes sets of critical level lines turn out to be unexpectedly complicated.

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