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ON THE INTEGER SOLUTIONS OF THE PELL EQUATION $x^2 = 17y^2 - 19^t$

Manju Somanath, J. Kannan, K. Raja and V. Sangeetha

Abstract

Let $d \neq 1$ be a positive non-square integer and N be any fixed positive integer. Then the equation $x^2 - dy^2 = \pm N$ is known as Pell's equation named after the famous mathematician John Pell. In this paper, we fix d and N to be two Coprimes 17 and 19 and search for non-trivial integer solutions to the equation $x^2 = 17y^2 - 19^t$, $t \in N$ for the different choices of t given by (i) $t = 1$, (ii) $t = 3$, (iii) $t = 5$, (iv) $2k$ and $t = 2k + 5$. Further, recurrence relations on the solutions are obtained.

Keywords and phrases: Pell equation, Brahma Gupta lemma, integer solutions, Diophantine equation.

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