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# HOPF BIFURCATIONS IN CANCER MODELS

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## Abstract

The purpose of the present paper is to prove that there are nonconstant periodic integral curves of a mathematical model of cancer growth. We shall prove, there are Hopf bifurcations in the mathematical model and that the oscillating solutions are uniformly, asymptotically stable.

**Keywords and phrases:** Hopf bifurcation, cancer, mass action kinetic system, the Brusselator.

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