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FUNCTIONAL EQUATION ORIGINATING FROM ARITHMETIC MEAN OF CONSECUTIVE TERMS OF AN ARITHMETIC PROGRESSION IS STABLE IN BANACH SPACE: DIRECT AND FIXED POINT METHOD

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Abstract

In this paper, the authors are proved the generalized Ulam-Hyers stability of a functional equation

$$f(y) = \frac{f(y+z) + f(y-z)}{2}$$

which is originating from arithmetic mean of consecutive terms of an arithmetic progression a fixed point approach. The application of the functional equation is also given.

Keywords and phrases: additive functional equations, Hyers-Ulam-Rassias stability.

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References

[1] J. Aczel and J. Dhombres, Functional Equations in Several Variables, Cambridge Univ. Press, 1989.

[2] T. Aoki, On the stability of the linear transformation in Banach spaces, J. Math. Soc. Japan 2 (1950), 64-66.

[3] M. Arunkumar, Solution and stability of Arun-additive functional equations, Internat. J. Math. Sci. Engng. Appl. 4(3) (2010), 33-46.

[4] D. G. Bourgin, Classes of transformations and bordering transformations, Bull. Amer. Math. Soc. 57 (1951), 223-237.

[5] L. Fenyö, Osservazioni Su alcuni teoremi di D. H. Hyers Istit Lombard, Accad. Sci. Leet. Rend. A 114 (1982), 235-242.

[6] L. Fenyö, On an inequality of P. W. Cholewa in General inequalities, 5, Internat Schrifenreiche Number. Math., Vol. 80, Birkhausern, Basel, 1987, pp. 277-280.

[7] Z. Gajda and R. Ger, Subadditive multifunctions and Hyers-Ulam stability, General Inequalites 5, Internat Schrifenreiche Number. Math., Vol. 80, Birkhausern, Basel, 1987.

[8] Z. Gadja, On stability of additive mappings, Internat. J. Math. Math. Sci. 14(3) (1991), 431-434.

[9] P. Gavruta, A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings, J. Math. Anal. Appl. 184 (1994), 431-436.

[10] D. H. Hyers, On the stability of the linear functional equation, Proc. Nat. Acad. Sci. 27 (1941), 222-224.

[11] D. H. Hyers, G. Isac and Th. M. Rassias, Stability of Functional Equations in Several Variables, Birkhäuser, Basel, 1998.

[12] S. M. Jung, Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis, Hadronic Press, Palm Harbor, 2001.

[13] Pl. Kannappan, Functional Equations and Inequalities with Applications, Springer Monographs in Mathematics, 2009.

[14] D. O. Lee, Hyers-Ulam stability of an additive type functional equation, J. Appl. Math. Comput. 13(1-2) (2003), 471-477.

[15] B. Margoils and J. B. Diaz, A fixed point theorem of the alternative for contractions on a generalized complete metric space, Bull. Amer. Math. Soc. 126(74) (1968), 305-309.

[16] V. Radu, The Fixed Point Alternative and the Stability of Functional Equations, Seminar on Fixed Point Theory Cluj-Napoca, Vol. IV, 2003, in press.

[17] J. M. Rassias, On approximation of approximately linear mappings by linear mappings, J. Func. Anal. 46 (1982), 126-130.

[18] Th. M. Rassias, On the stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc. 72 (1978), 297-300.

[19] Th. M. Rassias, On the stability of functional equations in Banach spaces, J. Math. Annal. Appl. 251 (2000), 264-284.

[20] K. Ravi and M. Arunkumar, On an n-dimensional additive functional equation with fixed point alternative, Proceedings of International Conference on Mathematical Sciences, Malaysia, 2007.

[21] K. Ravi, M. Arunkumar and J. M. Rassias, On the Ulam stability for the orthogonally general Euler-Lagrange type functional equation, Internat. J. Math. Sci. 3(8) (2008), 36-47.

[22] S. M. Ulam, Problems in Modern Mathematics, Chapter VI, Wiley, New York, 1960.