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EXISTENCE AND UNIQUENESS OF WEAK SOLUTION TO NONLINEAR PARABOLIC PROBLEM WITH HOMOGENEOUS NEUMANN BOUNDARY CONDITIONS INVOLVING IN SOBOLEV SPACE WITH VARIABLE EXPONENT

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Abstract

In this paper, we deal with the parabolic equation in $p(x)$- Laplacian form $b(u)_t - \Delta_{p(x)}u = f$ with homogeneous Neumann boundary conditions. We prove the existence and uniqueness of weak solution for $f \in L^{\infty}(Q)$ via time discretization technique by Euler forward scheme and some a-priori estimates. The functional setting involves Lebesgue and Sobolev spaces with variable exponent.

Keywords and phrases: parabolic equation, weak solution, semi-discretization.

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