JP Journal of Mathematical Sciences
Volume 23, Issues 1 & 2, 2018, Pages 1-10

EFFICIENT SIMULATION OF THE STOCHASTIC BURGERS EQUATION WITH ADDITIVE NOISE

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Abstract

This article is aimed to numerically validate the strong convergence rate for nonlinearitytruncated exponential Euler scheme, developed by Becker and Jentzen in [1] for abstract stochastic evolution equations, of stochastic Burgers equation with additive noise. Further, comparison of the proposed scheme is carried out with the simplified exponential Euler scheme, introduced in [25], to reveal the robustness of the considered scheme for stochastic Burgers equation with additive noise. Furthermore, pathwise convergence rate is also compared with the simplified exponential Euler scheme, defined above, to reveal the efficiency of the considered scheme.

Keywords and phrases: parabolic stochastic partial differential equations, strong convergence, pathwise approximation, stochastic Burgers equation, Turbulent flows.

Received March 8, 2019

References

[1] S. Becker and A. Jentzen, Strong convergence rates for nonlinearity-truncated Eulertype approximations of stochastic Ginzburg-Landau equations, Stoch. Proc. Appl. 129(1) (2019), 28-69.

[2] E. R. Benton and G. W. Platzman, A table of solutions of the one dimensional Burgers equation, Q. Appl. Math. 10(196) (1972), 166-173.

[3] J. M. Burgers, A mathematical model illustrating the theory of turbulence, Adv. Appl. Mech. 1 (1948), 171-199.

[4] S. Keleti and X. B. Reed, Spectral properties of exact random solutions to Burgers equation for modified Thomos initial conditions, Comput. Fluids 16(2) (1988), 147-173.

[5] P. G. Saffman, Lectures on homogenous turbulence, topics in non-linear physics, Munich. (1966).

[6] I. Hosokawa and K. Yamamoto, Turbulence in the randomly forced one dimensional Burgers flow, J. Stat. Phys. 13(3) (1975), 245-272.

[7] D. H. Chambers, R. J. Adrian, P. Moin, D. S. Srewart and H. J. Sung, Karhunen-Loéve expansion of Burgers' model of turbulence, Phys. Fluids 31(9) (1988), 2573-2582.

[8] H. Choi, R. Temam, P. Moin and J. Kim, Feedback control for unsteady flow and its application to the stochastic Burgers equation, J. Fluid Mech. 253 (1993), 509-543.

[9] Dah-Teng Jeng, Forced model equation for turbulence, Phys. Fluids 12(10) (1969), 2006-2010.

[10] D. H. Chambers, Ph. D. Thesis, University of Illinois at Urbana-Champaign, 1987.

[11] J. L. Lions, Optimal Control Systems Goverened by Partial Differential Equations, Springer, 1971.

[12] F. Flandoli and D. Gatarek, Martingale and stationary solutions for stochastic Navier-Stokes equations, Probab. Theo. Related Fields 102 (1995), 367-391.

[13] G. D. A. Prato and A. Debussche, Stochastic Cahn-Hilliard equation, Nonlinear Anal. 26 (1996), 241-263.

[14] A. Cox and E. Hausenblas, A perturbation result for quasi-linear stochastic differential equations in UMD Banach spaces, preprint (2012), arXiv:1203.1606.

[15] D. Blömker, F. Flandoli and M. Romito, Markovianity and ergodicity for a surface growth PDE, Ann. Probab. 37 (2009), 275-313.

[16] G. D. A. Prato, A. Debussche and R. Temam, Stochastic Burgers' equation, BIT Numer. Math. NoDEA 1 (1994), 389-402.

[17] E. Hausenblas, Numerical analysis of semilinear stochastic evolution equations in Banach spaces, J. Comput. Appl. Math. 147 (2002), 485-516.

[18] M. Kamrani and S. M. Hosseini, Spectral collocation method for stochastic Burgers equation driven by additive noise, Math. Comput. Simul. 82 (2012), 1630-1644.

[19] G. J. Lord and J. Rougemont, A numerical scheme for stochastic PDEs with Gevrey regularity, IMA J. Numer. Anal. 24 (2004), 587-604.

[20] G. J. Lord and T. Shardlow, Postprocessing for stochastic parabolic partial differential equations, SIAM J. Numer. Anal. 45 (2007), 870-889.

[21] T. Müller-Gronbach and K. Ritter, An implicit Euler scheme with non-uniform time discretization for heat equations with multiplicative noise, BIT 47 (2007), 393-418.

[22] T. Müller-Gronbach and K. Ritter, Lower bounds and nonuniform time discretization for approximation of stochastic heat equations, Found. Comput. Math. 7 (2007), 135-181.

[23] T. Müller-Gronbach, K. Ritter and T. Wagner, Optimal pointwise approximation of a linear stochastic heat equation with additive space-time white noise in Monte Carlo and Quasi-Monte Carlo Methods 2006, Springer, Berlin, 2007, pp. 577-589.

[24] T. Müller-Gronbach, K. Ritter and T. Wagner, Optimal pointwise approximation of infinite-dimensional Ornstein-Uhlenbeck process, Stochastic Dynam. 8 (2008), 519-541.

[25] A. Jentzen, P. E. Kloeden and G. Winkel, Efficient simulation of nonlinear parabolic SPDEs with additive noise, Ann. Appl. Probab. 21(3) (2011), 908-950.

[26] D. Blömker, M. Kamrani and S. M. Hosseini, Full discretization of the stochastic Burgers equation with correlated noise, IMA J. Numer. Anal. 33(3) (2013), 825-848.