ASSET ALLOCATION UNDER A LOGARITHMIC UTILITY FUNCTION WITH REGIME SWITCHING

Fengxia Hu

Abstract

Under a logarithmic utility function, this paper studies optimal portfolio problem in a discrete-time, finite-horizon setting, where short-selling and leveraging are prohibited. We suppose that the random return of risky asset depends on the economic environments which are described by a Markov chain. Employing dynamic programming theory, we obtain a closed form solution of the optimal investment strategy. In addition, with the help of stochastic orders, we discuss the properties of the optimal investment strategy, and then investigate the impact of economic environment regimes on the optimal strategy. Finally, we derive the order of ranking the optimal proportions invested in risky asset.

Keywords and phrases: Stochastic orders, Markov chain, optimal investment strategy, logarithmic utility function, dynamic programming.

Received July 29, 2017; Revised September 19, 2017

References

- [1] P. P. Boyle and H. L. Yang, Asset allocation with time variation in expected returns, Insurance: Math. Econom. 21 (1997), 201-218.
- [2] K. C. Cheung and H. L. Yang, Asset allocation with regime-switching: discrete-time case, Astin Bull. 34(1) (2004), 99-111.
- [3] D. Duffie and R. Kan, A yield-factor model of interest rates, Math. Fin. 6 (1996), 379-406.
- [4] R. J. Elliott and J. Hinz, Portfolio analysis, hidden Markov models and chart analysis by PF-diagrams, Internat. J. Theo. Appl. Fin. 5 (2002), 385-399.
- [5] R. J. Elliott and P. E. Kopp, Mathematics of Financial Markets, 2nd Ed., Springer, 2004.

- [6] R. J. Elliott and J. van der Hoek, An application of hidden Markov models to asset allocation problems, Finance Stochas. 3 (1997), 229-238.
- [7] R. J. Elliott, L. L. Chan and T. K. Siu, Option pricing and Esscher transform under regime switching, Ann. Finan. 1(4) (2005), 423-432.
- [8] R. J. Elliott, W. C. Hunter and B. M. Jamieson, Financial signal processing, Internat. J. Theo. Appl. Finan. 4 (2001), 567-584.
- [9] X. Guo, Information and option pricings, Quantitative Finan. 1 (2001), 38-44.
- [10] J. D. Hamilton, A new approach to the economic analysis of nonstationary time series and the business cycle, Econometri. 57(2) (1989), 357-384.
- [11] H. Markowitz, Portfolio selection, J. Finan. 7 (1952), 77-91.
- [12] R. C. Merton, Lifetime portfolio selection under uncertainty: the continuous time case, Review Econom. Statist. 51(3) (1969), 247-257.
- [13] R. C. Merton, Optimal consumption and portfolio rules in a continuous-time model, J. Econo. Theo. 3 (1971), 373-413.
- [14] S. R. Pliska, Introduction to Mathematical Finance, United States, Blackwell Publishing, 1997.
- [15] M. Shaked and J. G. Shanthikumar, Stochastic Orders, Springer, 2007.
- [16] L. R. Sotomayor and A. Cadenillas, Explicit solutions of consumption investment problems in financial markets with regime switching, Math. Fin. 19(2) (2009), 251-279.
- [17] G. Yin and X. Y. Zhou, Markowitz mean-variance portfolio selection with regime switching: from discrete-time models to their continuous-time limits, IEEE Trans. Automatic Contr. 49 (2003), 349-360.
- [18] X. Y. Zhou and G. Yin, Markowitz mean-variance portfolio selection with regime switching: a continuous-time model, SIAM J. Contr. Optim. 42 (2004), 1466-1482.