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A HIGH-ORDER MODIFIED METHOD FOR THE BACKWARD HEAT CONDUCTION PROBLEM WITH A CONVECTION TERM

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Abstract

In this paper, we consider a backward heat conduction problem (BHCP) with a convection term in a strip. By adding a higher-order $(4k+1$ order) partial differential term, we obtain an approximate stable solution to the BHCP. Error estimates between exact and approximate solutions are given, which reach the Hölder and logarithmic type stability estimation respectively under different priori information by selecting the appropriate regularization parameters. The higher-order modified method is an effective method of regularization to solve the BHCP. At the end, we give an example of numerical simulations.

Keywords and phrases: backward heat conduction problem, higher-order modification, ill-posed, regularization.

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