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SPATIO-TEMPORAL MATHEMATICAL MODELING OF INFECTIOUS DISEASES WITH CROSS DIFFUSION EFFECTS

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Abstract

In this paper, we study analytically a class of nonlinear parabolic reaction-diffusion systems modeling the spread of infectious diseases with cross-diffusion terms. This model is governed by an S-I-R type system. First, we prove the global existence of weak solution to this class of system by means of an approximation process, the Faedo-Galerkin method, some a priori estimates and compactness arguments. Then, using Gronwall's lemma, we establish an existence and uniqueness result of weak solution for this class of systems without the cross-diffusion terms.

Keywords and phrases: infectious diseases, S-I-R model, cross-diffusion system, weak solutions, Faedo-Galerkin.

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