

BENFORD OR NOT-BENFORD? HOW TO TEST FOR THE FIRST-DIGIT-LAW

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Abstract

In this paper, we discuss several methods used to examine the goodness-of-fit of a given dataset to the so-called Benford's law. While the use of distance measures itself suffer from theoretical founded critical values, the procedures used in literature so far do not provide a more clear statement. Apart from using graphical methods (like histogram, etc.), which are not part of the discussion, the often used Distortion Factor Model is extended to the Mantissa-Distortion-Factor (MDF) and a Benford-specific test procedure is introduced. This test is based the property of invariance of scale and base, which is an underlying feature of the First-Digit-Law. The so called Transformation-Invariance-Test (TIT) is derived as a closed form test-procedure. The practical power of the TIT is shown using a widely used macroeconomic dataset.

Keywords and phrases: Newcomb-law, goodness-of-fit, quality of datasets, invariance.

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