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HIGH-ORDER ACCURATE NUMERICAL SCHEME FOR NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract

A fourth-order numerical method is proposed for solving first-order nonlinear integro-differential equations. The method is based on finite difference approximation of derivatives and an unconventional quadrature approximation of integrals. The unconventional quadrature scheme emanates from approximating the leading error term of the conventional trapezoid quadrature rule. The nonlinear non-homogeneous parts are discretely interpolated. This resulted to a fully nonlinear algebraic system which is approximated using a nonlinear solver. The proposed method is tested on two nonlinear non-homogeneous equations with known exact solutions. Numerical solutions are observed for convergence, order of accuracy and appearance of non-physical oscillations. The results show that the method is convergent, has fourth-order of accuracy, and produce no non-physical oscillations for all mesh sizes - which is a numerical attestation of the stability of the proposed method.

Keywords and phrases: nonstandard trapezoid rule, finite difference scheme, nonlinear Fredholm equation, experimental order of convergence, manufactured solutions.

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